**The Laws and Properties of Exponents**

| Operation | Action | Law/Property | Example(s) |
|---|---|---|---|
| Multiplication | Add exponents of the same base | $a^m a^n = a^{m+n}$ | $x^3 x^4 = x^7$ |
| Division | Subtract exponents of the same base | $\frac{a^m}{a^n} = a^{m-n}$ | $\frac{x^7}{x^3} = x^4$ |
| Power to a Power | Multiply exponents | $(a^m)^n = a^{mn}$ | $(x^4)^3 = x^{12}$ |
| Power to a Power of a Product | Multiply exponents | $(ab)^n = a^n b^n$ | $(3x^2)^3 = 3^3 x^6 = 27x^6$ |
| Power to a Power of a Quotient | Multiply exponents | $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ | $\left(\frac{y}{x}\right)^3 = \frac{y^3}{x^3}$ |
| Zero Exponent | | $a^0 = 1,\ a \neq 0$ | $5^0 = 1,\ (2x^3y^2)^0 = 1$ |
| Negative Exponents | Take reciprocal of the base and change the sign of exponent to positive | $a^{-n} = \frac{1}{a^n}$<br>$\frac{a^{-n}}{b^{-n}} = \frac{b^n}{a^n}$<br>$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$ | $x^{-3} = \frac{1}{x^3}$<br>$\frac{x^{-3}}{y^{-4}} = \frac{y^4}{x^3}$<br>$\left(\frac{x}{y}\right)^{-3} = \left(\frac{y}{x}\right)^3 = \frac{y^3}{x^3}$ |

**Simplify with only positive exponents.**

1. $(3x^3)(4x^5)$

2. $\frac{x^5y^3}{x^2y}$

3. $(x^3y^2)^4$

4. $(2x^3y^2)^4$

5. $\left(\frac{2x^3}{3y^2}\right)^3$

6. $(5x^0y^3)^2$

7. $\frac{x^{-5}y^4}{w^{-3}}$

8. $\left(\frac{3x^2}{y^3}\right)^{-3}$

9. $(x^5)(3x^2)(4x^7)$

10. $(2x^4y^{-3})^{-2}$

11. $\frac{(3y^3)(2y^2)^3}{(y^4)^3}$

12. $\left(\frac{3x^5y^4}{x^0y^{-3}}\right)^{-2}$

**The Laws and Properties of Radicals**

In $\sqrt[n]{a}$ , $n$ is the index.

| Operation | Law | Example | Law | Example |
|---|---|---|---|---|
| Product Same index | $\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{a}$ | $\sqrt{50} = \sqrt{25} \cdot \sqrt{2} = 5\sqrt{2}$ | $\sqrt[n]{a}\sqrt[n]{a} = \sqrt[n]{ab}$ | $\sqrt{2} \cdot \sqrt{3} = \sqrt{6}$ |
| Quotient Same index | $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ | $\sqrt[3]{\frac{5}{8}} = \frac{\sqrt[3]{5}}{\sqrt[3]{8}} = \frac{\sqrt[3]{5}}{2}$ | $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$ | $\frac{\sqrt{200}}{\sqrt{2}} = \sqrt{\frac{200}{2}} = \sqrt{100} = 10$ |
| Radical within a Radical | $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$ | $\sqrt{\sqrt[3]{64}} = \sqrt[6]{64} = 2$ | | |

Properties of $\sqrt[n]{a}$ , where $n$ is a positive integer.

| Property | Examples when the index equals the exponent | |
|---|---|---|
| **(1)** $\left(\sqrt[n]{a}\right)^n = a$ if $\left(\sqrt[n]{a}\right)$ is a real number | $\left(\sqrt{5}\right)^2 = 5$ | $\left(\sqrt[3]{-8}\right)^3 = -8$ |
| **(2)** $\sqrt[n]{a^n} = a$ if $a \geq 0$ | $\sqrt{5^2} = 5$ | $\sqrt[3]{2^3} = 2$ |
| **(3)** $\sqrt[n]{a^n} = a$ if $a < 0$ and $n$ is odd | $\sqrt[3]{(-2)^3} = -2$ | $\sqrt[5]{(-2)^5} = -2$ |
| **(4)** $\sqrt[n]{a^n} = \lvert a\rvert$ if $a < 0$ and $n$ is even | $\sqrt{(-3)^2} = \lvert -3\rvert = 3$ | $\sqrt[4]{(-2)^4} = \lvert -2\rvert = -2$ |

**Additional Examples**
**Simplify each radical. Assume all variables represent positive real numbers.**

**Example 1a:** Using Perfect Squares

$\sqrt{75}$
$\sqrt{75} = \sqrt{25} \cdot \sqrt{3} = 5\sqrt{3}$

**Example 1b:** Using Prime Factorization

$\sqrt{75}$
$\sqrt{75} = \sqrt{5^2} \cdot \sqrt{3} = 5\sqrt{3}$

**Example 2a:** Using Perfect Cubes

$\sqrt[3]{320}$
$\sqrt[3]{320} = \sqrt[3]{64 \cdot 5} = \sqrt[3]{64} \cdot \sqrt[3]{5} = 4\sqrt[3]{5}$

**Example 2b:** Using Prime Factorization

$\sqrt[3]{320}$
$\sqrt[3]{320} = \sqrt[3]{2^6 \cdot 5} = \sqrt[3]{2^6} \cdot \sqrt[3]{5} = 4\sqrt[3]{5}$

**Example 3:**

$\sqrt{3x^2y^3} \cdot \sqrt{6x^5y}$
$\sqrt{3x^2y^3} \cdot \sqrt{6x^5y} = \sqrt{18x^7y^4} = 3x^3y^2\sqrt{2x}$

**Simplify, remove radicals if at all possible. Assume all variables represent positive real numbers.**

1. $\sqrt{98}$

2. $\sqrt{\dfrac{3}{4}}$

3. $\sqrt{\dfrac{27}{36}}$

4. $\sqrt{16x^8}$

5. $\sqrt[3]{125}$

6. $\sqrt[3]{108}$

7. $\sqrt[5]{-64}$

8. $\sqrt[4]{81x^5y^8}$

9. $\sqrt[3]{16xy^2} \cdot \sqrt[3]{3x^2y^2}$

**In problems 10-15 determine the exponent needed to remove the radical symbol**

10. $\sqrt{5} \cdot \sqrt{5^?} = 5$

11. $\sqrt[7]{4^2} \cdot \sqrt[7]{4^?} = 4$

12. $\sqrt[5]{3^2} \cdot \sqrt[5]{3^?} = 3$

13. $\sqrt[8]{x^3} \cdot \sqrt[8]{x^?} = x$

14. $\sqrt[3]{y} \cdot \sqrt[3]{y^?} = y$

15. $\sqrt[5]{z^4} \cdot \sqrt[5]{z^?} = z$

**Multiplication of Polynomials**

**Example 1:** Distributive Property $5x(2x-3)$

$5x(2x-3)=10x^2-15x$

**Example 2:** Multiplying binomials $(4x+5)(3x-2)$

$(4x+5)(3x-2)=12x^2-8x+15x-10=12x^2+7x-10$

**Product Formulas**

| Formula | Example |
|---|---|
| $(x+y)(x-y)=x^2-y^2$ or $(x-y)(x+y)=x^2-y^2$ | $(x+5)(x-5)=x^2-25$ or $(x-5)(x+5)=x^2-25$ |
| $(x+y)^2=x^2+2xy+y^2$ | $(2x+3)^2=(2x)^2+2(2x)(3)+(3)^2=4x^2+12x+9$ |
| $(x-y)^2=x^2-2xy+y^2$ | $(2x-3)^2=(2x)^2-2(2x)(3)+(3)^2=4x^2-12x+9$ |
| $(x+y)^3=x^3+3x^2y+3xy^2+y^3$ | $(2x+3)^3=(2x)^3+3(2x)^2(3)+3(2x)(3)^2+(3)^3$ $=8x^3+36x^2+54x+27$ |
| $(x-y)^3=x^3-3x^2y+3xy^2-y^3$ | $(2x-3)^3=(2x)^3-3(2x)^2(3)+3(2x)(3)^2-(3)^3$ $=8x^3-36x^2+54x-27$ |

**Multiply, use product formulas when they apply.**

1. $3x(2x^2+5x-3)$

2. $(x+5)(x+7)$

3. $(3x+4)(2x-7)$

4. $(x-3)^2$

5. $(x+5)^2$

6. $(3x-2)^2$

7. $(x+3)^3$

8. $(2x+5)^3$

9. $(x-4)^3$

10. $(3x-1)^3$

**Factoring Polynomials**

**Example 1:** Greatest Common Factor $15x^3 - 25x$

$15x^3 - 25x = 5x(3x^2 - 5)$

Examples 2 through 4 are Trinomial Factoring of $ax^2 + bx + c$ where $a = 1$

**Example 2:** Factors of Same Signs $x^2 + 7x + 12$

$x^2 + 7x + 12 = (x + \quad)(x + \quad)$

$x^2 + 7x + 12 = (x + 3)(x + 4)$

**Example 3:** Factors of Same Signs $x^2 - 7x + 12$

$x^2 + 7x + 12 = (x - \quad)(x - \quad)$

$x^2 + 7x + 12 = (x - 3)(x - 4)$

**Example 4:** Factors of Different Signs $x^2 - 2x - 15$

$x^2 - 2x - 15 = (x - \quad)(x + \quad)$

$x^2 - 2x - 15 = (x - 5)(x + 3)$

**Examples 5:** Trinomial Factoring of $ax^2 + bx + c$ where $a \neq 1,\ a \neq 0$

| | |
|---|---|
| Factors of Same Signs | $6x^2 + 19x + 10$ |
| Factors of $a \cdot c$ with sum of 19 | $6 \cdot 10 = 60$, $4 + 15 = 19$ |
| Rewrite trinomial as | $6x^2 + 19x + 10 = 6x^2 + 4x + 15x + 10$ |
| Group as two binomials | $6x^2 + 19x + 10 = (6x^2 + 4x) + (15x + 10)$ |
| Factor Greatest Common Factor | $6x^2 + 19x + 10 = 2x(3x + 2) + 5(3x + 2)$ |
| Factor Greatest Common Factor | $6x^2 + 19x + 10 = (3x + 2)(2x + 5)$ |

**Factoring Formulas**

| Name | Formula | Example |
|---|---|---|
| Difference of Two Squares | $x^2 - y^2 = (x + y)(x - y)$ | $9x^2 - 16 = (3x)^2 - (4)^2 = (3x + 4)(3x - 4)$ |
| Difference of Two Cubes | $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$ | $8x^3 - 27 = (2x)^3 - (3)^3 = (2x - 3)[(2x)^2 + (2x)(3) + (3)^2]$ <br> $= (2x - 3)(4x^2 + 6x + 9)$ |
| Sum of Two Cubes | $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$ | $8x^3 + 27 = (2x)^3 + (3)^3 = (2x + 3)[(2x)^2 - (2x)(3) + (3)^2]$ <br> $= (2x + 3)(4x^2 - 6x + 9)$ |

**Problems for Factoring Polynomials**

**Factor each completely.**

1. $21x+14$

2. $x^2+12x+35$

3. $x^2-10x+24$

4. $x^2-4x-21$

5. $x^2-3x-40$

6. $x^2+13x+30$

7. $x^2-1$

8. $25x^2-49y^2$

9. $2x^2-98$

10. $6x^2+13x+5$

11. $3x^2-11x+10$

12. $12x^2+x-6$

13. $x^3-8$

14. $x^3-125$

15. $x^3+27$

16. $8x^3+125$

**Fractional Expressions**

A fractional expression is a quotient of two algebraic expressions. As in any fraction, the denominator may not equal zero, division by zero is not permitted. A special case of fractional expressions is the quotient of two polynomials, which is referred to as a rational expression.

Follow these three steps in simplifying fractional expressions and rational expressions.

| | |
|---|---|
| Step 1 | Factor the numerator and the denominator if possible |
| Step 2 | Determine any values that would make the denominator equal to zero. |
| Step 3 | Remove any factors of one, factors that are the same in the numerator and denominator. |

**Example 1:** Simplify the fractional expression and determine any restricted values.

$$\frac{x^2+3x-10}{x^2-25}$$

Step 1 Factor

$$\frac{x^2+3x-10}{x^2-25}=\frac{(x+5)(x-2)}{(x+5)(x-5)}$$

Step 2 Determine Restricted Values $x \neq \pm 5$

Step 3 Remove Factors of One

$$\frac{x^2+3x-10}{x^2-25}=\frac{(x+5)(x-2)}{(x+5)(x-5)}=\frac{(x-2)}{(x-5)}$$

**Example 2:** Simplify the fractional expression and determine any restricted values.

$$\frac{3x^2-5x-2}{x^2+3x-10}$$

Step 1 Factor

$$\frac{3x^2-5x-2}{x^2+3x-10}=\frac{(3x+1)(x-2)}{(x+5)(x-2)}$$

Step 2 Determine Restricted Values $x \neq -5,\ x \neq 2$

Step 3 Remove Factors of One

$$\frac{3x^2-5x-2}{x^2+3x-10}=\frac{(3x+1)(x-2)}{(x+5)(x-2)}=\frac{(3x+1)}{(x+5)}$$

**Problems for Fractional Expressions**

**Simplify each if possible and state any restrictions.**

1. $\dfrac{(x+2)(x-4)}{(x+2)(x+5)}$

2. $\dfrac{5x+20}{x^2+10x+24}$

3. $\dfrac{x^2+13x+30}{x^2-9}$

4. $\dfrac{x^2+12x+35}{x^2-3x-40}$

5. $\dfrac{x^2-49}{x^2+14x+49}$

6. $\dfrac{x^3-125}{x^2-25}$